

*The Physics of Cataclysmic Variables and Related Objects*  
*ASP Conference Series, Vol. XXX, 2002*  
*B.T. Gänsicke, K. Beuermann, K. Reinsch*

## Ab initio Simulations of Accretion Disc Boundary Layers

Philip J. Armitage

*University of St Andrews, School of Physics and Astronomy, North  
 Haugh, Fife KY16 9SS, UK*

**Abstract.** I discuss the results of simplified three dimensional magnetohydrodynamic simulations of the boundary layer between a disc and a non-rotating, unmagnetized star. Strong magnetic fields, possibly approaching equipartition with the thermal energy, occur in the boundary layer due to the shearing of disc-generated fields. The mean boundary layer magnetic field, which is highly variable on an orbital timescale, is estimated to exceed  $\sim 50$  kG for a CV with an accretion rate of  $10^{-9} M_{\odot} \text{yr}^{-1}$ . However, these fields do not drive efficient angular momentum transport within the boundary layer. As a consequence the radial velocity in the boundary layer is low, and the density high.

### 1. Introduction

An accretion flow onto a weakly magnetic star must make a transition between the angular velocity in the disc, which is usually close to Keplerian, and the stellar angular velocity, which is typically much smaller. The boundary layer across which this transition occurs can be important both energetically, releasing up to half the bolometric luminosity of the accretion flow, and as a site of variability, in accretion onto protostars, white dwarfs and neutron stars.

The aim of this contribution is to present the results of preliminary magnetohydrodynamic (MHD) simulations of disc boundary layers (Armitage 2002)<sup>1</sup>. First though, it is useful to summarize the basics of boundary layer theory (e.g. Frank, King, & Raine 1992). To do so, let us forget magnetic fields for the time being, and assume that angular momentum transport in the accretion flow can be described as a viscosity, which depends upon the local shear in the angular velocity  $\Omega$ . In the disc,  $d\Omega/dr < 0$ , and angular momentum is transported outwards. If the star is slowly rotating, the inflowing gas inevitably reaches a maximum angular velocity  $\Omega_{\text{max}}$ , as illustrated in Figure 1. At the radius  $r_{\text{max}}$  of this maximum, there is zero torque, so that the rate of accretion of angular momentum is simply  $r_{\text{max}}^2 \Omega_{\text{max}} \dot{M}$ . Inside this radius, we reach the boundary layer region, where  $d\Omega/dr > 0$ . Angular momentum transport here is inward, and acts to mix the accreted angular momentum into the star.

A somewhat more quantitative picture of the structure of the boundary layer follows from considering the radial component of the momentum equation.

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<sup>1</sup>See also animations at <http://star-www.st-and.ac.uk/~pja3/movies.html>

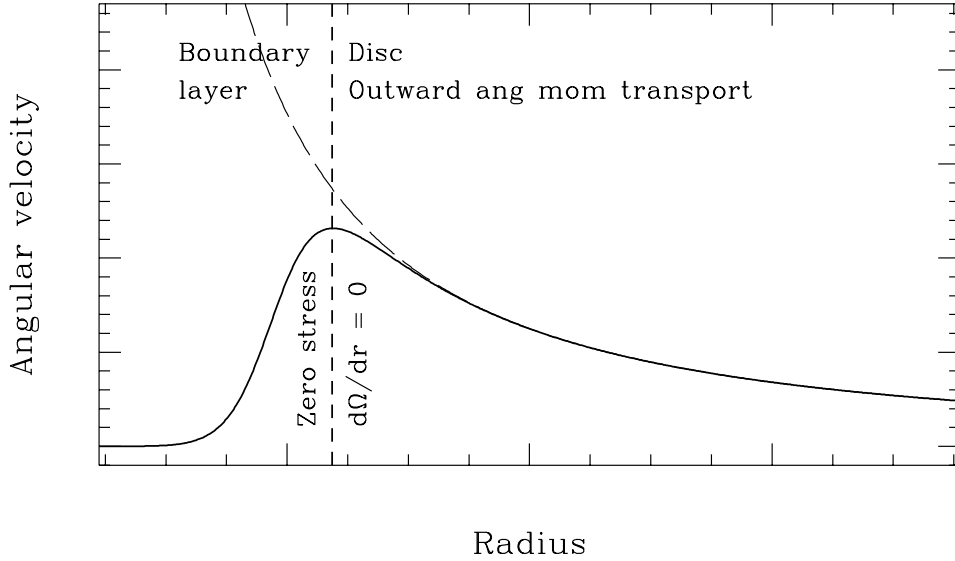


Figure 1. Illustration of the boundary layer structure for a geometrically thin accretion flow in which angular momentum transport can be described as a viscosity. In the disc, outward angular momentum transport leads to mass inflow, and the angular velocity is close to Keplerian (dashed curve). Close to the star, the angular velocity reaches a maximum. There is no viscous stress in the disc at this radius. At smaller radii, inward angular momentum transport within the boundary layer determines the boundary layer structure.

In a steady state, and ignoring viscous and magnetic terms,

$$v_r \frac{\partial v_r}{\partial r} - \frac{v_\phi^2}{r} + \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{GM}{r^2} = 0,$$

where the symbols have their conventional meanings. In a geometrically thin disc the centrifugal term  $v_\phi^2/r$  accurately balances the gravitational force  $GM/r^2$ , but this is not the case in the boundary layer. If the boundary layer has radial width  $\delta r_{\text{BL}}$ , there are evidently two possibilities:

- Pressure forces balance gravity. Writing  $P = \rho c_s^2$ , where  $c_s$  is the sound speed, we have  $\rho^{-1} \partial P / \partial r \sim c_s^2 / \delta r_{\text{BL}}$ . If the boundary layer, like the disc, is to be geometrically thin, then  $c_s \ll v_K$ , the Keplerian orbital velocity. Pressure gradients can only balance gravity if  $\delta r_{\text{BL}} \ll r_*$ .
- The  $v_r \partial v_r / \partial r \sim v_r^2 / \delta r_{\text{BL}}$  term balances gravity. Causality demands that the radial velocity be smaller than the sound speed (Pringle 1977), so  $v_r^2 \ll v_K^2$  and again we conclude that  $\delta r_{\text{BL}} \ll r_*$ .

This analysis then suggests that the boundary layer will be radially narrow, *unless* the temperature becomes high enough to raise the sound speed to of

the order of the Keplerian orbital velocity. Real calculations show that radially narrow boundary layers are expected in cataclysmic variables (Popham & Narayan 1995), whereas broader boundary layers are possible in rapidly accreting pre-main-sequence stars such as FU Orionis objects (Popham 1996; Kley & Lin 1999), and around neutron stars (Popham & Sunyaev 2001). A broad dynamical boundary layer has the interesting property that it allows accretion to occur without spinning up the star, even at stellar rotation rates well below break up (Popham & Narayan 1991).

To go beyond these general considerations, it is necessary to know something about the angular momentum transport in the boundary layer, since this will determine the radial velocity and density structure. The traditional choice in the *disc* is to express the viscosity using the Shakura & Sunyaev (1973)  $\alpha$  prescription,

$$\nu = \alpha c_s h$$

where  $h$  is the disc scale height and  $\alpha$  is a constant. Unfortunately this proves disastrous if extended into the boundary layer, since it can lead to unphysical supersonic radial velocities. To remedy this, alternatives to the  $\alpha$  prescription have been proposed, which have the effect of reducing  $\nu$  in the boundary layer. A simple example is to replace the vertical scale height  $h$  in the above equation with the (smaller) radial pressure scale height  $h_r$ , though more elaborate schemes have also been devised (Lynden-Bell & Pringle 1974; Papaloizou & Stanley 1986; Popham & Narayan 1992).

## 2. The role of magnetic fields

How might magnetic fields affect this picture? One possibility is that magnetic fields, generated in the disc by magnetorotational instabilities (Balbus & Hawley 1991), could be amplified by shear in the boundary layer (Pringle 1989). The basic argument is that the rate of shearing in the boundary layer exceeds that in the disc by a factor  $\sim r_*/\delta r_{\text{BL}}$ , which is large, perhaps 10-100. Unless the loss processes for the magnetic field (for example Parker instability, or reconnection) are similarly accelerated, the result will be a strong, predominantly toroidal field. Pringle (1989) estimated that the resultant Alfvén speed in the boundary layer could be of the order of,

$$v_A \sim \left(\frac{h}{r}\right)^{1/2} v_K,$$

though a contemporary estimate would be somewhat lower since MHD turbulence in discs leads to poloidal fields that are substantially smaller than the toroidal component. Nonetheless, substantial field amplification appears possible. I note that analogous processes have been considered in the context of black hole accretion (Krolik 1999), where the shear arises in the dynamically unstable region interior to the marginally stable orbit.

Such field amplification does not depend upon the existence of magnetic instabilities or dynamo processes in the boundary layer, and is therefore a rather robust prediction. The origin and magnitude of angular momentum transport in the boundary layer is, however, an interesting question in its own right. The

identification of magnetorotational instabilities (MRI) as the key ingredient for disc angular momentum transport leaves the situation in the boundary layer unclear. The condition for the MRI to operate is that,

$$\frac{d\Omega^2}{dr} < 0,$$

which is crucially less stringent than the Rayleigh criteria for hydrodynamic instability,  $d(r^2\Omega)/dr < 0$ . As a result, discs are always MRI unstable. Fields in the boundary layer, however, would be stable by this criteria. This has two implications. First, it suggests that the viscosity in the boundary layer might well be expected to be reduced as compared to the disc. This is promising, as we have already noted that existing models *require* such a reduction in the efficiency of angular momentum transport. Second, it means that fundamentally different processes must be responsible for angular momentum transport in the boundary layer. This is not so good, as it undermines the various prescriptions for boundary layer viscosity and suggests that existing quantitative models for the boundary layer are likely to be of limited validity.

### 3. Numerical simulations

The ZEUS code (Stone & Norman 1992a, 1992b, Norman 2000) is used to solve the equations of ideal MHD. ZEUS is an explicit, Eulerian MHD code, which uses an artificial viscosity to capture shocks. The simulations model the inner disc, boundary layer, and hydrostatic envelope of the star, and are fully three-dimensional. Simplifications are made by assuming an isothermal equation of state, and by ignoring the vertical component of gravity in the disc structure. Simulating an unstratified disc reduces the computational burden, and should be a reasonable approximation close to the equatorial plane. The simulation geometry is cylindrical,  $(z, r, \phi)$ , using uniform gridding in the vertical and azimuthal directions, and a grid in the radial direction which concentrates resolution close to the boundary layer. The highest resolution calculation discussed here used 480 radial, 90 azimuthal, and 60 vertical grid cells.

The initial conditions for the simulation comprise a static, non-rotating and unmagnetized atmosphere, surrounded by a Keplerian disc with an approximately gaussian density profile. The disc is initially seeded with a weak vertical magnetic field to initiate magnetorotational instabilities. A boundary layer then forms automatically once the MRI is active and mass in the disc is flowing inwards. The boundary conditions are periodic in  $z$  and  $\phi$ , reflecting at  $r = r_{\text{in}}$  ( $v_r = B_r = 0$ ), and set to outflow at  $r = r_{\text{out}}$ .

### 4. Results

Figure 2 shows the strength of the magnetic fields generated towards the end of the simulation, when the MRI in the disc has reached a saturated and quasi-steady state. The azimuthal domain is  $45^\circ$  across, and the grid extends from an inner edge at  $r_{\text{in}} = 1$  (in arbitrary units) to  $r_{\text{out}} = 5$ . The innermost region is occupied by high density, pressure supported gas, which is initially unmagnetized

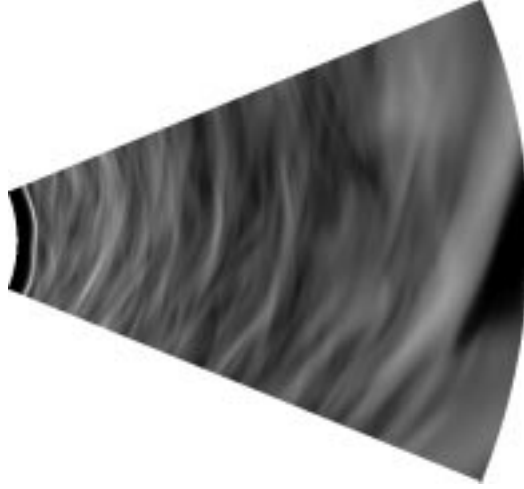


Figure 2. Map of the magnetic field strength as a fraction of the thermal energy in the simulation, integrated vertically. The boundary layer is the narrow bright stripe of field near the inner edge of the disc. The radial grid in the simulation is much finer at small  $r$  to concentrate resolution in the boundary layer region.

and remains so during the course of the run. Outside this is a narrow, strongly magnetized boundary layer, which appears as a bright stripe in the magnetic field map. This region is resolved across 20-30 grid points in the highest resolution run so far completed. An animation of the evolution shows that the strong field region in the boundary layer is highly variable, buckling and changing in strength on a dynamical timescale. At larger radii the gas in the accretion disc is in close to Keplerian rotation, and displays the complex pattern of magnetic fields seen in previous MHD simulations of disc turbulence.

#### 4.1. Magnetic field strength

Figure 3 shows the time-averaged strength of the magnetic fields generated in the simulation, both in terms of the absolute magnetic field energy density (in arbitrary code units), and as a fraction  $\beta^{-1}$  of the thermal energy in the disc. The expected amplification of disc magnetic fields by shear in the boundary layer is clearly seen, with the magnetic field energy in the boundary layer exceeding that in the disc immediately outside by around an order of magnitude. The same conclusion applies – albeit less strikingly – if the magnetic energy is compared to the thermal energy. In the disc, the energy in the magnetic fields generated in the simulation is only around 5% of the thermal energy. In the boundary layer, this rises to around 15%.

Figure 3 also shows how the magnetic fields vary with time. Fields in the boundary layer are highly variable – even more so than in the disc. This makes it hard to judge, given the limited period of evolution simulated, whether the resolution is sufficient to have achieved numerical convergence in the boundary layer. Pending higher resolution runs, the field strengths quoted above are therefore probably lower limits.

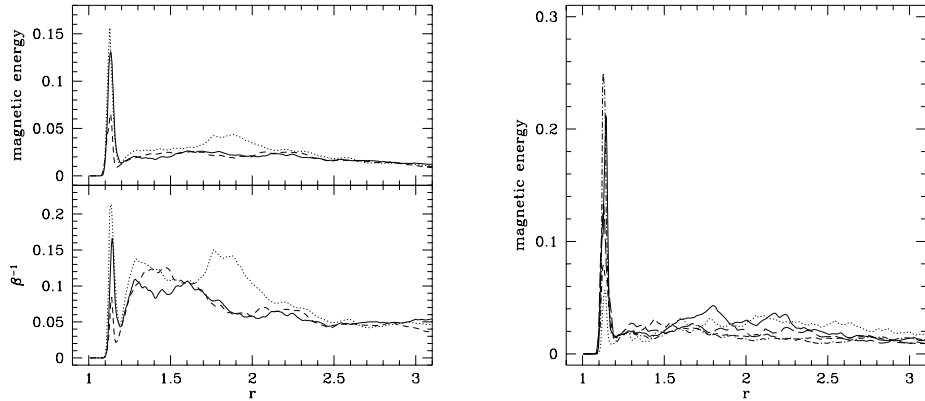


Figure 3. Strength of the magnetic fields in the disc and boundary layer. The lefthand plot shows the mean (averaged over several independent timeslices) magnetic energy density in three simulations with different resolution, both in absolute terms (upper panel) and as a fraction of the thermal energy (lower panel). The righthand plot shows slices from one simulation at different times. There are large fluctuations in the boundary layer field strength.

#### 4.2. Angular momentum transport

Figure 4 shows the angular and radial velocities obtained in the simulations, again averaged over several timeslices. The radial velocity is everywhere highly subsonic, and is largest in the disc just outside the boundary layer. The small radial velocity in the boundary layer occurs because angular momentum transport in this region is inefficient, and as a result the boundary layer occupies relatively high density gas (right panel of the Figure) adjacent to the stellar envelope. Analysis of the magnetic stress as a function of radius suggests that the stress acts like a viscosity in so far as it vanishes very close to the radius where  $d\Omega/dr = 0$ . In the boundary layer itself, the magnetic stress is very small.

This lack of angular momentum transport in the boundary layer implies a different boundary layer structure as compared to one-dimensional models. For the rather thin boundary layer simulated here, however, the differences may not be large, since all existing boundary layer viscosity prescriptions imply a sharp reduction in the efficiency of angular momentum transport in a narrow boundary layer. More substantial differences may occur for boundary layers of greater radial extent.

#### 4.3. Scaling to observed systems

The current simulations are too rudimentary to be able to provide a self-contained measure of the boundary layer field strength in real systems. However, we can make an order of magnitude estimate by making use of the scaling relations for the central density  $\rho$  and temperature  $T_c$  of  $\alpha$  discs. For a Kramers' opacity

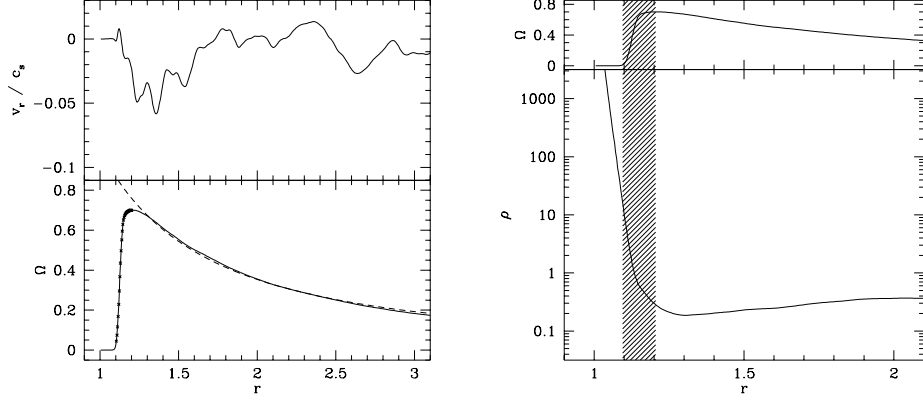


Figure 4. Left panel: the mean radial and angular velocity in the simulation. The radial velocity in the boundary layer is low, so the density (right panel) in the boundary layer is relatively high.

( $\kappa \propto \rho T_c^{-7/2}$ ) Frank, King & Raine (1992) quote,

$$\begin{aligned} \rho &= 3.1 \times 10^{-8} \alpha^{-\frac{7}{10}} \left( \frac{\dot{M}}{10^{16} \text{ gs}^{-1}} \right)^{\frac{11}{20}} \left( \frac{M}{M_\odot} \right)^{\frac{5}{8}} \left( \frac{r}{10^{10} \text{ cm}} \right)^{-\frac{15}{8}} f^{\frac{11}{5}} \text{ gcm}^{-3} \\ T_c &= 1.4 \times 10^4 \alpha^{-\frac{1}{5}} \left( \frac{\dot{M}}{10^{16} \text{ gs}^{-1}} \right)^{\frac{3}{10}} \left( \frac{M}{M_\odot} \right)^{\frac{1}{4}} \left( \frac{r}{10^{10} \text{ cm}} \right)^{-\frac{3}{4}} f^{\frac{6}{5}} \text{ K} \\ f &= \left[ 1 - \left( \frac{r_*}{r} \right)^{\frac{1}{2}} \right]^{\frac{1}{4}}. \end{aligned}$$

Appropriate values in the inner regions of CV discs are a radius of  $\approx 10^9$  cm, a white dwarf mass of  $0.6M_\odot$ , and an  $\alpha$  parameter of  $\sim 0.1$ .

With these scaling relations in hand, we can estimate the gas pressure in the disc at  $r = 10^9$  cm, near the boundary layer. The simulations can then be used to estimate first the magnetic energy in the disc, and from that the magnetic energy in the boundary layer. The results suggest that the magnetic energy density in the boundary layer is roughly comparable to the thermal energy in the inner disc, which implies,

$$B_{\text{BL}} \simeq 50 \left( \frac{\dot{M}}{10^{-9} M_\odot \text{ yr}^{-1}} \right)^{17/40} \text{ kG}.$$

Given the limitations of the simulations this figure is probably a lower limit to the true field strength, and is definitely subject to considerable uncertainty. It should be regarded as an order of magnitude estimate only.

## 5. Summary

This contribution has discussed three-dimensional MHD simulations of accretion disc boundary layers. The main results of the work so far are,

- Angular momentum transport in the boundary layer is inefficient. This means that the radial velocity in the boundary layer is highly subsonic, and the density high. Although this is qualitatively consistent with the modified viscosity prescriptions used in one-dimensional models of the boundary layer, such prescriptions have no physical basis, and results derived from them need to be treated with caution.
- Strong magnetic fields are generated in the boundary layer from shearing of disc fields. For weakly magnetized stars, the strongest magnetic fields will be in the boundary layer.
- The magnetic fields and boundary layer generally are highly variable.

Further simulations, incorporating some aspects of the thermal structure of the boundary layer, are needed to investigate these questions in more detail.

**Acknowledgments.** My thanks to Andrew Cumming, Kristen Menou and Gordon Ogilvie for stimulating discussions during the course of the meeting. Computations made use of the UK Astrophysical Fluids Facility.

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